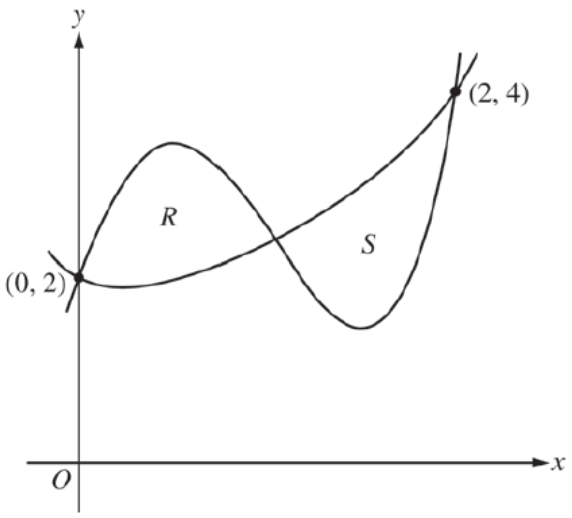
AP Calculus AB

Unit 7 - Volume

1. **(Calculator Permitted)** Find the average value of $f(x)=x\sin x$ on the interval $[1,\pi]$. Show the set-up to your solution, then use your calculator. Your answer should be correct to three decimal places.

2. 2015 – AB #2 **(Calculator Permitted)**



. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

(a) Find the sum of the areas of regions R and S.

3. The base of a solid in the *xy*-plane is bounded by the *x*-axis, the *y*-axis and the line $y=-x+2$. Cross sections of the solid perpendicular to the *x*-axis are squares. Find the volume.

4. The base of a solid is bounded in the first quadrant by the *x*-axis, the *y*-axis and the graph of $y=\sqrt{9-x^2}$. Cross sections of this solid perpendicular to the *x*-axis are semicircles. Find the volume of the solid.

5. **(Calculator Permitted)** The base of a solid is the region in the first quadrant bounded by the graphs of $y=e^{-x^2}$, $y=1-\cos x$ and the *y*-axis. For this solid, each cross section perpendicular to the *x*-axis is a square. Find the volume of the solid.

6. **(Calculator Permitted)** The base of a solid is the region in the first quadrant bounded by the graphs of $y=\sqrt{x}$, $y=e^{-3x}$ and the vertical line $x=1$. For this solid each cross section perpendicular to the *x*-axis is a rectangle whose height is 5 times the length of its base. Find the volume of the solid.

7. $\int \frac{x}{\sqrt{9-x^2}}dx =$

A $-\frac{1}{2}\ln\sqrt{9-x^2}+C$ B $\sin^{-1}\frac{x}{3}+C$ C $-\sqrt{9-x^2}+C$

D $-\frac{1}{4}\sqrt{9-x^2}+C$ E $2\sqrt{9-x^2}+C$

8. **(Calculator Permitted)** Find the instantaneous rate of change of $f(x)=\dfrac{x}{\sin x}$ when $x=\dfrac{\pi}{3}$.

9. The curve of $y=\dfrac{2x^2}{4-x^2}$ has:

A two vertical asymptotes
B two horizontal asymptotes and one vertical asymptote
C two vertical but not horizontal asymptotes
D one horizontal and one vertical asymptote
E one horizontal and two vertical asymptotes

10. 2014 AB #1 **(Calculator Permitted)**

Grass clippings are placed in a bin, where they decompose. For $0\le t\le 30$ the amount of grass clippings remaining in the bin is modeled by $A(t)=6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0\le t\le 30$. Indicate units of measure.

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

Selected Answers:

3. $\dfrac{8}{3}$	4. $\dfrac{9\pi}{4}$	5. 0.461	6. 1.554
7. C	8. 0.456 or 0.457	9. E	

AP Calculus AB - Worksheet 60 **Volumes of Solids with Known Cross Sections**

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1. **(Calculator Permitted)** The region bounded by the *y*-axis and the graphs of $y = \dfrac{x^3}{1+x^2}$ and $y = 4 - 2x$ is the base of a solid. For this solid, each cross-section perpendicular to the *x*-axis is a square. Find the volume of this solid.

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2. **(Calculator Permitted)** The region in the first quadrant bounded by the graphs of $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$ is the base of a solid. For this solid, the cross sections perpendicular to the *x*-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

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3. The base of a solid is bounded by $y = x^3, y = 0,$ and $x = 1$. Find the volume of the solid that has cross sections that are squares taken perpendicular to the **y-axis**. (Hint: This is a 'dy' problem)

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4. If $f(x) = x \ln x,$ then $f'''(e) =$ A $\dfrac{1}{e}$ B 0 C $-\dfrac{1}{e^2}$ D $\dfrac{1}{e^2}$ E $\dfrac{2}{e^3}$

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5. Set up, but do not evaluate, an expression involving one or more integrals that can be used to find the area bounded by the parabola $y = x^2$ and the lines $y = 1$ and $y = 9$.

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6. $\lim\limits_{x \to 3} \dfrac{x+3}{x^2 - 9} =$ A $+\infty$ B 0 C $\dfrac{1}{6}$ D $-\infty$ E Nonexistent

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7. If $F(x) = 4 + \int_0^x (3t^2 + 2t + 1)\,dt,$ find $F(3)$. (This is called an integral function)

Answers:

1. 8.997	2. 0.077 or 0.078	3. $\dfrac{1}{10}$	4. C
5. $\dfrac{104}{3}$	6. E		

AP Calculus AB - Worksheet 61 **Volumes by Rotation**

Let *R* be the region enclosed by the given functions. Find the volume of the solid generated by revolving *R* about the given axis.

(Calculator Permitted on 1-4)

1. $y = x^3,\ x = 2,\ y = 0$ (x-axis)

2. $y = 2x,\ y = 0,\ x = 2$ $(\text{revolve about } y = -1)$

3. $y = \frac{x}{2},\ x = 0,\ y = 2$ $(\text{revolve about } y = 3)$

4. $y = \sqrt{x},\ x = 0,\ y = 2$ (x-axis)

(No Calculator Permitted on 5-8)

5. $y = x^2, x = -1,\ x = 4,\ y = 0$ (x-axis)

6. $y = x^4,\ x = 1,\ y = 0$ (x-axis)

7. $y = \sqrt{9 - x^2},\ y = 0$ (x-axis)

8. $y = x^2, y = 2x$ $(\text{revolve around the line } y = 4)$

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9. **(Calculator Permitted)** Calculate the area bounded by $y = x^3 - 4x + 4$ and $y = 3x - 2$.

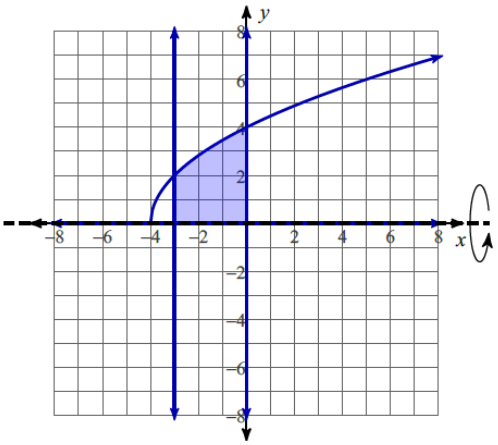
Answers:

1. $\frac{128\pi}{7}$ units^3	2. $\frac{56}{3}\pi$	3.	4. 8π units^3	5. 205π
6. $\frac{\pi}{9}$	7. 36π	8. $\frac{32\pi}{5}$		

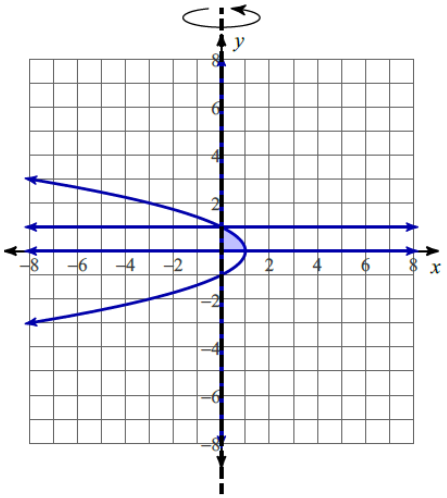
AP Calculus AB - Worksheet 62 Volumes of Solids (Rotation)

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the the given axis. Set up, but do not evaluate the integral.

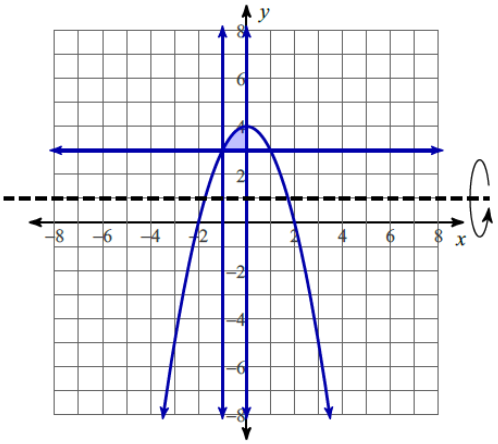
1) $y = 2\sqrt{x+4}$, $y = 0$, $x = -3$, $x = 0$
Axis: $y = 0$



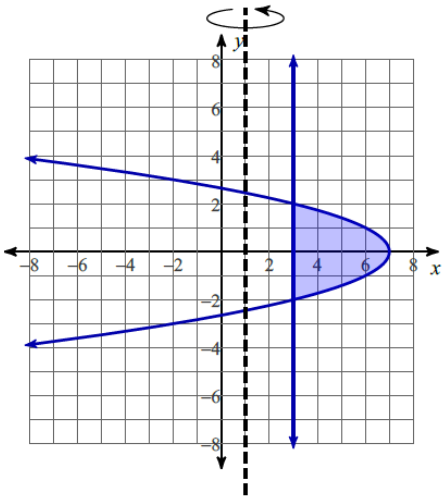
2) $x = -y^2 + 1$, $x = 0$, $y = 0$, $y = 1$
Axis: $x = 0$



3) $y = -x^2 + 4$, $y = 3$, $x = -1$, $x = 0$
Axis: $y = 1$

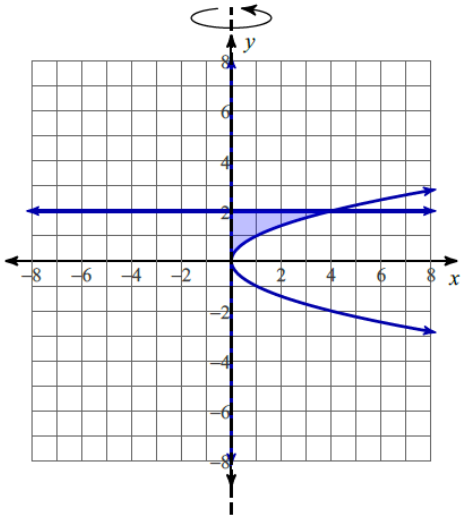


4) $x = -y^2 + 7$, $x = 3$
Axis: $x = 1$

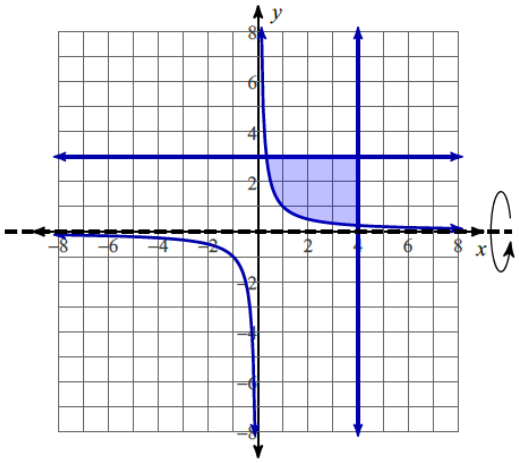


For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the given axis.

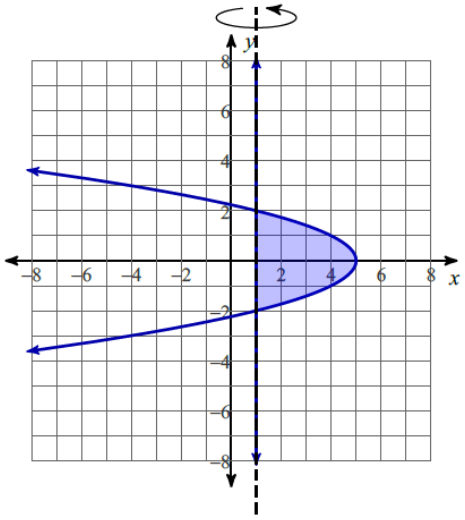
5) $x = y^2,\ x = 0,\ y = 2$
Axis: $x = 0$



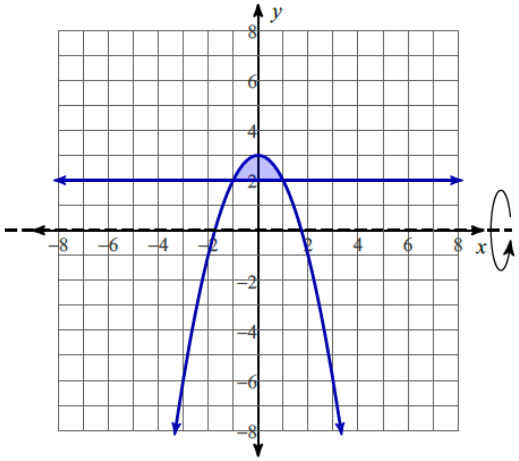
6) $y = 3,\ y = \frac{1}{x},\ x = 4$
Axis: $y = 0$



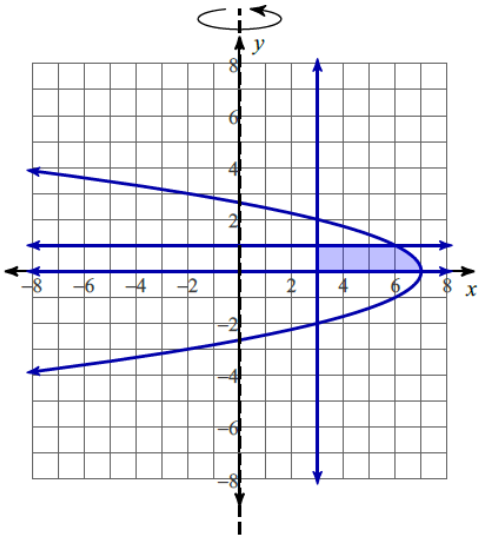
7) $x = -y^2 + 5,\ x = 1$
Axis: $x = 1$



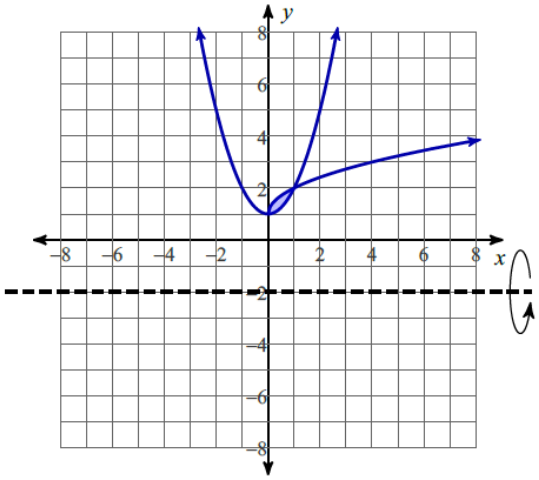
8) $y = -x^2 + 3,\ y = 2$
Axis: $y = 0$



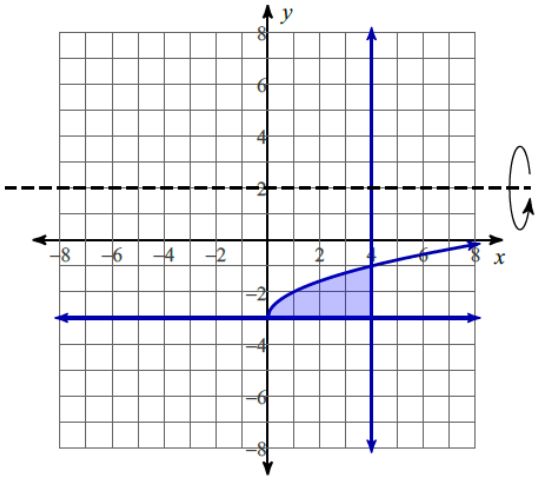
9) $x = -y^2 + 7,\ x = 3,\ y = 0,\ y = 1$
Axis: $x = 0$



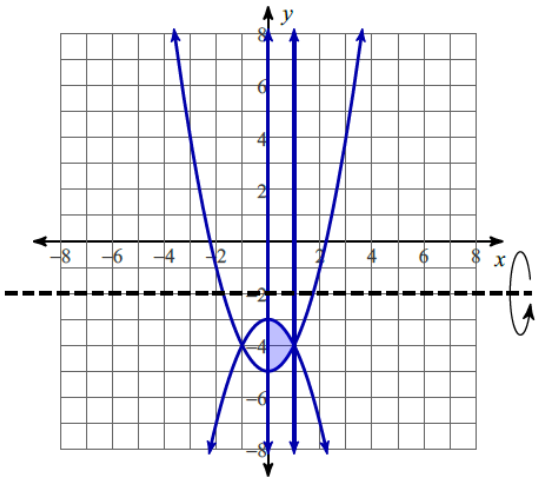
10) $y = \sqrt{x} + 1,\ y = x^2 + 1$
Axis: $y = -2$



11) $y = -3,\ y = \sqrt{x} - 3,\ x = 4$
Axis: $y = 2$



12) $y = x^2 - 5,\ y = -x^2 - 3,\ x = 0,\ x = 1$
Axis: $y = -2$



Answers to

1) $\pi\int_{-3}^{0}\left(2\sqrt{x+4}\right)^2\,dx$

2) $\pi\int_{0}^{1}\left(-y^2+1\right)^2\,dy$

3) $\pi\int_{-1}^{0}\left(\left(-x^2+3\right)^2-2^2\right)\,dx$

4) $\pi\int_{-2}^{2}\left(\left(-y^2+6\right)^2-2^2\right)\,dy$

5) $\frac{32}{5}\pi \approx 20.106$

6) $\frac{121}{4}\pi \approx 95.033$

7) $\frac{512}{15}\pi \approx 107.233$

8) $\frac{32}{5}\pi \approx 20.106$

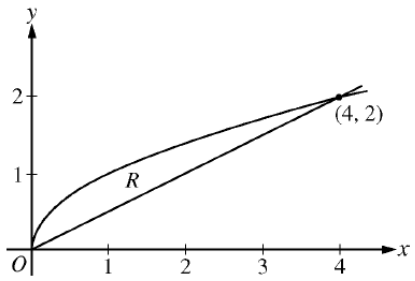
9) $\frac{533}{15}\pi \approx 111.631$

10) $\frac{23}{10}\pi \approx 7.226$

11) $\frac{136}{3}\pi \approx 142.419$

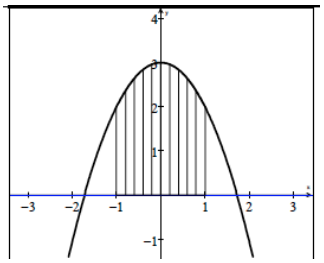
12) $\frac{16}{3}\pi \approx 16.755$

AP Calculus AB - Worksheet 63 **Volumes – Rotation and Cross-Sectional Area**

1	Find the **volume** of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $y=2$ and $x=0$ about: a) the *x*-axis b) the *y*-axis c) the line $y=2$ d) the line $x=4$
2.	 Let R be the region bounded by the graphs of $y=\sqrt{x}$ and $y=\frac{x}{2}$, as shown in the figure above. a) Find the AREA of *R*. b) Find, but do not evaluate, an integral expression that can be used to find the volume of the solid generated by rotating the region *R* about the horizontal line $y=2$. c) The region *R* is the base of solid. Cross sections taken perpendicular to the *x*-axis are squares. Write, but do not evaluate, an integral expression that can be used to find the volume of this solid.
3	Given: $\int_0^4 f(x)dx=7;\ \int_0^9 f(x)dx=12;\ \int_4^9 g(x)dx=-2;\ \int_0^4 g(x)dx=8$ Find each of the following: *a)* $\int_0^9 g(x)dx$ *b)* $\int_4^9 f(x)dx$ *c)* $\int_9^4 g(x)dx$ *d)* $\int_4^9 [f(x)+3g(x)]dx$ *e)* $\int_9^9 g(x)$
4.	$\lim_{x\to 0}\frac{\sin 4x}{2x}$
5.	Find the volume of the solid whose base is the region bounded between the curves $y=x$ and $y=x^2$, and whose cross sections perpendicular to the *x*-axis are squares.
6	The base of a certain solid is the region enclosed by $y=\sqrt{x}, y=0,$ and $x=4$. Every cross section perpendicular to the *x*-axis is a semicircle with its diameter across the base. Find the volume of the solid.
7	Consider the region enclosed between $y=\sqrt{x},\ x=4,$ and the *x*-axis. Find the volume of the solid that is formed when the enclosed region is revolved about the **y-axis**.

Answers:

1. $a) 8\pi$ $b) \frac{32\pi}{5}$ $c) \frac{8\pi}{3}$ $d) \frac{224\pi}{15}$	2. a) $\frac{4}{3}$ b) $\pi\int_0^4\left(\left(2-\frac{x}{2}\right)^2-\left(2-\sqrt{x}\right)^2\right)dx$ c) $\int_0^4\left(\left(\sqrt{x}-\frac{x}{2}\right)^2\right)dx$	3. a) 6 b) 5 c) 2 d) -1 e) 0	4. 2	5. $\frac{1}{30}$	6. π	7.



1. A region is enclosed by the graphs of $y = 3 - x^2$ and the vertical lines $x = -1$ and $x = 1$ as show in the figure above.
   a. Find the area of the enclosed region.
   b. Find the volume of the solid that is generated by revolving the enclosed region about the *x*-axis. **Do not simplify your final answer!**
   c. Write, but do not evaluate, an integral expression that can be used to find the volume of the solid that is generated by revolving the enclosed region about the line $y = 4$.
   d. The shaded region is the base of a solid. For this solid, cross sections taken perpendicular to the *x*-axis are semicircles. Write, but do not evaluate, an integral expression that can be used to find the volume of this solid.

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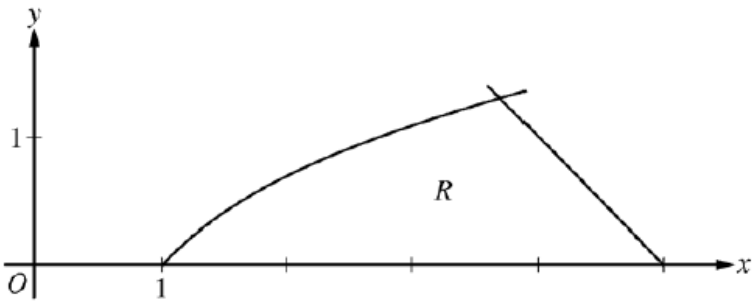
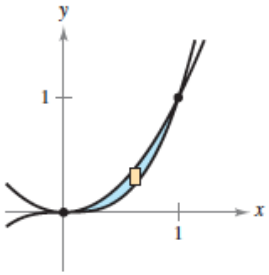
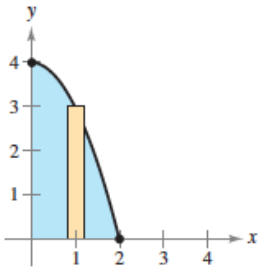


2. Let R be the region in the first quadrant bounded by the *x*-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.
   (a) Find the area of R.
   (b) Region R is the base of a solid. For the solid, each cross section perpendicular to the *x*-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
   (c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.



3. A region is enclosed by the graphs of $y = x^2$ and $y = x^3$ as shown in the figure above. Find the volume of the solid that is generated by revolving the region about the *x*-axis. Simplify your final answer.

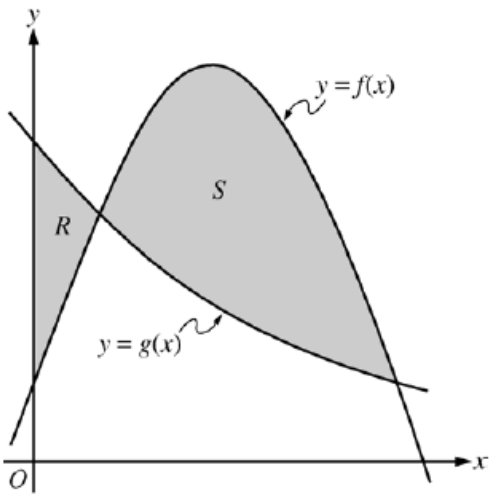


4. A region is enclosed by the graphs of $y = 4 - x^2$, $y = 0$ and $x = 0$ as show in the figure above.
   a. Find the area of the enclosed region.
   b. Find the volume of the solid that is generated by revolving the enclosed region about the *x*-axis. **Do not simplify your final answer!**
   c. Write, but do not evaluate, an integral expression that can be used to find the volume of the solid that is generated by revolving the enclosed region about the line $y = 6$.
   d. The base of a solid is the enclosed region. Write, but do not evaluate, an integral expression that can be used to find the volume of the solid if cross sections taken perpendicular to the *x*-axis are isosceles right triangles with on leg across the region.

5. A region is enclosed by the graphs of $y = \sqrt{x}$, the *x*-axis and the line $x = 4$. Write, but do not evaluate, the integral expressions that can be used to find the volume of the solid that is generated by revolving the region about the given line. (You do not need to simplify).
   a. the *x*-axis.
   b. the line $y = -2$.
   c. the *y*-axis.
   d. the line $x = 6$.

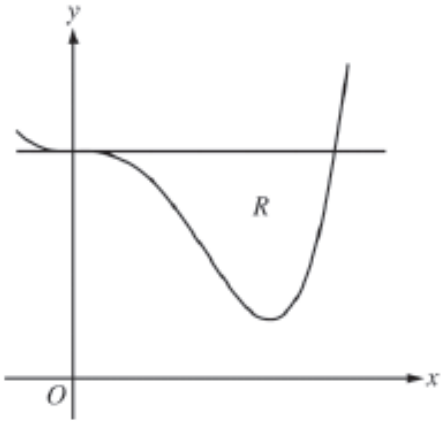
AP Calculus AB – Worksheet 65 **Area and Volume Review – AP Questions**

Calculator



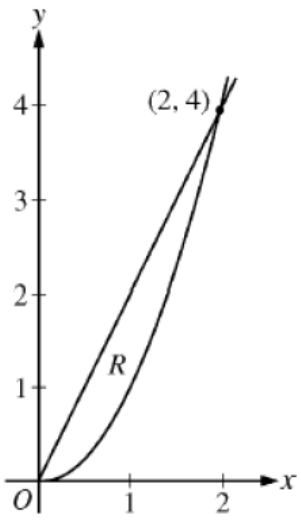
1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the area of S.
   (c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

Calculator



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.
   (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
   (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.
   (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

No Calculator



3. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.